Bayesian Representations and Information Metrics for Cognitive Radar

Nathan A. Goodman
University of Oklahoma, Advanced Radar Research Center & School of ECE
3190 Monitor Avenue
Norman, OK 73019
UNITED STATES

goodman@ou.edu

ABSTRACT

We investigate fundamental ideas and philosophies behind the optimization of radar measurements for completion of desired tasks. In contrast to traditional approaches, where waveforms are design for "good" ambiguity functions, search areas are uniformly illuminated, and radar operations are executed in a pre-defined manner, cognitive radar admits the possibility of adapting its illumination characteristics in real time in order to achieve tasks with fewer resources such as power, spectrum use, and timeline. However, using a radar in this way requires techniques for representing features or events of interest, and objective functions for optimizing the parameters of the illumination. In this paper, we take a high-level view of radar measurement optimization for different applications with the goal of encouraging the reader to consider radar measurement in a more flexible, less restrictive, manner.

1.0 INTRODUCTION

Traditional radar systems, and sensors in general, are typically designed to capture a one-for-one mapping of the observed scene. In other words, sensors are designed to obtained high-fidelity, unambiguous mappings of range, Doppler, angle, wavelength, or whichever parameters a particular sensor can measure. The sensor response is typically designed to be an approximately shift-invariant pixel function that captures localized *resolution cells* over a region of interest; hence, there must be at least one measurement per resolution cell. The highest fidelity representation is obtained when the pixel-like imaging function has narrow width (i.e., high resolution for minimal blurring) and low sidelobes (i.e., reduced leakage across resolution cells). Consequently, this approach leads to increasing requirements for high resolution and low sidelobes, as quantified by sensing metrics such as the radar ambiguity function, array patterns, optical point spread functions, or similar imaging function.

This traditional approach to sensor design and development takes a *capture-everything* approach. If a one-to-one, high-fidelity mapping of the entire observed parameter space can be obtained, then in some sense, the data set is complete and various exploitations tasks can be pursued. These exploitation tasks are critical, as sensor data is rarely taken simply for the sake of collecting data, but rather for the purpose of some goal such as detection, characterization, and/or parameter estimation. However, much of the collected data is non-informative for a given exploitation task or set of tasks, which means that sensing resources were expended to collect data that are not useful to the task(s) at hand. In many cases, available prior knowledge could have been used to predict that certain parts of the data would be non-informative, but cannot be acted on because the sensor is designed to operate in a rigid, prescribed fashion to collect complete representations, regardless of whether prior knowledge indicates a waste of resources. We suggest here that such rigid sensing paradigms do not adequately treat sensing resources such as size, weight, power (SWAP), and cost as finite and in short supply.

An alternative approach to sensor design and operation is to consider whether the information in a set of measurements can be increased, thereby making more efficient use of finite sensing resources. In this paradigm, the actual measurements collected by the sensor should be informed by the intended exploitation tasks, as well as available prior information (including information obtained from other recent measurements). The sensor operation may even become adaptive, because past measurements will inform current and future measurements in order to improve their utility. When we develop representations for tasks of interests, noise, interference, and parameters that aren't of interest, then these representations can be updated as measurements are received and can be used to optimize the parameters and structure of future measurements. In this paper, we refer to such closed-loop, adaptive sensing as cognitive sensing (and as cognitive radar when these concepts are specifically applied to radar). In some cases the data obtained by a cognitive radar are non-traditional, such that there is an increase in computational complexity needed to achieve an exploitation tasks. However, trends in improved computing capability suggest that the added costs of computation may, at least in the long run, be small compared to the benefits of reduced SWAP in the sensing hardware.

In this paper, we explore concepts of adaptive sensing and sensing resource allocation through canonical, textbook-type inference problems. We consider an information-based model for representing exploitation tasks and nuisance parameters, and use the standard inference problems to show how the information-based model can lead to interesting conclusions regarding sensing resource efficiency and use. We consider signal detection, detection with a nuisance parameter, detection of multiple signals, and continuous parameter estimation. We also describe some of the challenges in optimizing multiple simultaneous tasks through a joint detection and range estimation example. The goal of this paper is that the reader will begin to think more broadly with respect to how sensors are designed and operated, with hopefully a desire to work toward more flexible, agile, and efficient radar systems in the future.

2.0 VIRTUAL SOURCE MODEL FOR INFERENCE TASKS

The idea of a virtual source model was first presented in [1]. The authors in [1] recognized that the Shannon mutual information (MI) [2] inherent in a sensing task is not the same as the information needed to reconstruct an entire data set. In particular, they considered an optical imaging problem, and noted that despite the uncertainty associated with all the pixels in an image, the maximum entropy associated with a target detection problem is only 1 bit; hence, for a detection task there is a maximum of 1 bit of information that can be obtained through the sensing process. Of course, this bit cannot be observed directly, and many measurements may need to be made to deal with nuisance parameters such as unknown target location. But despite all the measurements to be made, the ultimate decision simplifies to the answer of a binary yes/no question with a worse-case prior uncertainty of 1 bit.

The virtual source model can be explained using Figure 1. In any inference problem, there is a truth about the sensing environment that we wish to know – for example, detection of a target or a parameter (e.g., range) of a signal of interest. This truth is generated from a random source according to a pdf that is consistent to the task and represents the distribution of truth values that would be observed over many trials of the inference task. Defining a realization of this source variable as Θ, then the source variable has either a probability mass function for discrete decision tasks or a probability density function (pdf) for a continuous parameter estimation problem.

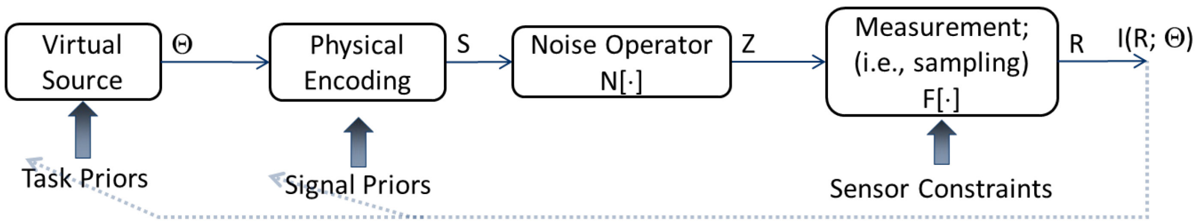

Figure 1: Virtual source model for Bayesian representation of a sensing inference problem.

For now, we denote this distribution as $p(\Theta)$ for both types (discrete versus continuous) of tasks. We cannot observe a particular realization of Θ directly, but instead we observe a signal that is an encoding of Θ as manifested in a physical quantity. For example, if Θ is a range value to be estimated, then we observe a signal that is delayed according to the physical propagation time. The observed signal can also be modified by various nuisance parameters; for example, the signal might have a Doppler shift in addition to its range delay. If the exploitation task is to estimate the target range, then Doppler shift is a nuisance parameter that effects the observed signal and makes the desired inference problem more difficult. In another example, Θ could represent a binary detection task, but if the target is present, then nuisance parameters might be unknown range and Doppler shift. We define the distribution of observed physical signals incident on the sensor and conditioned on the true source parameter as $p(S|\Theta)$. The observed physical signals *S* are usually corrupted by noise, such that the noisy signal *Z* is the one that is conditioned and sampled by the sensing hardware. We define the obtained measurements as *R*, such that the information gained through the sensing process, with respect to the task that is encoded in the sensing variable Θ, is $I(R;\Theta)$.

3.0 INFERENCE TASK AND MEASUREMENT OPTIMIZATION EXAMPLES

3.1 Detection of a Known Signal

The simplest detection problem is the detection of a known signal in noise. We begin with this simple problem to demonstrate the virtual source model and the relationships between probability of detection, probability of miss, and information gained.

For a detection problem, we let the virtual source variable be a binary random variable taking on the values 0 or 1. If the target is absent, then Θ = 0. If the target is present, then Θ = 1. Therefore, we can express the physical signal incident on the sensor as

$$z(t) = \Theta s(t) + n(t), \tag{1}$$

such that the signal to be measured consists of only noise if the target is absent or a known signal plus noise if the target is present. We assume that the noise is additive white Gaussian noise (AWGN), and because the signal is known, the detection problem can be reduced to the calculation of a scalar-valued sufficient statistic obtained by correlating $z(t)$ with $s^*(t)$ [3]. For radar, we usually implement Neyman-Pearson detection, whereby the sufficient statistic is compared to a threshold that provides a pre-defined probability of false alarm [3]. Here, we take a Bayesian approach, which we acknowledge is non-standard for radar detection problems and requires knowledge of the prior probabilities of target presence. In this example, we assume that both hypotheses are equally likely, yielding a maximum information of 1 bit.

Figure 2 shows probability of detection and probability of miss for detection of a known signal in noise with the Bayesian approach. The different curves correspond to different detection thresholds applied to the posterior probability of target presence. For example, the prior probability of target presence is 0.5. This probability is updated using Bayes' theorem and the received measurement to obtain a posterior probability, which is compared to a threshold. Obviously, the higher the threshold, the lower the probability of detection. Figure 3 shows the MI gained as a function of SNR for the same detection problem. If the posterior probability of target presence is $\tilde{P}_1 = \Pr\{\Theta = 1 | z(t)\}$, then the MI computed on a given simulation trial is

$$MI = 1 - \left(-\tilde{P}_1 \log_2 \tilde{P}_1 - \left(1 - \tilde{P}_1\right)\log_2\left(1 - \tilde{P}_1\right)\right). \quad (2)$$

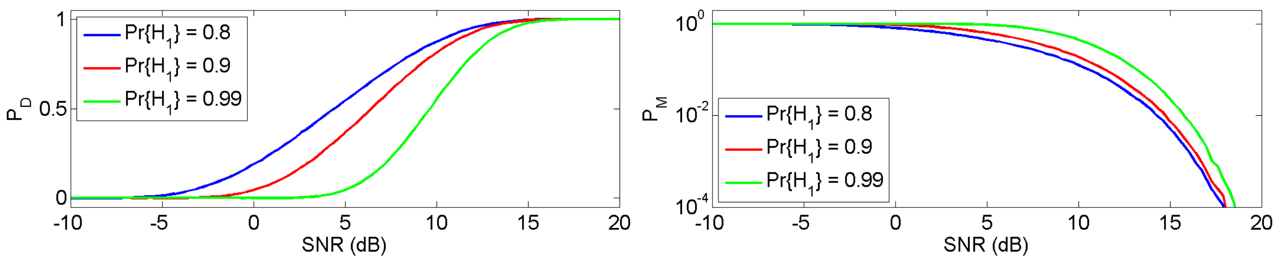

Figure 2: Probability of detection (left) and probability of miss (right) for a detection problem.

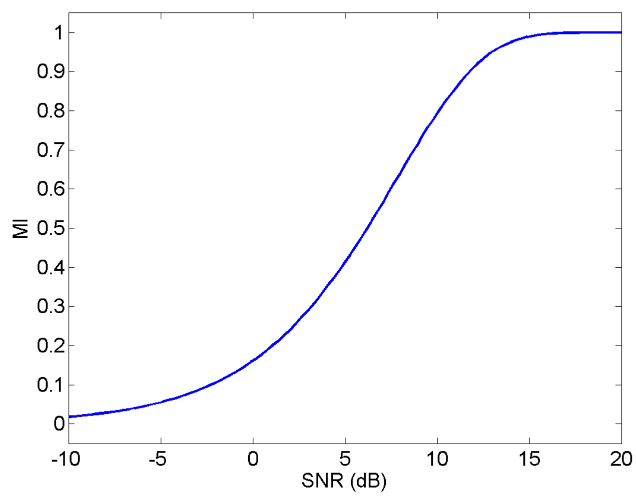

Figure 3: Information gained versus SNR for detection of a known signal in noise.

The MI result shows that there is diminished benefit, from an information perspective, in increasing SNR above approximately 15 dB. The probability of miss result shows that the probability of miss is decreasing (when shown in log scale) with SNR above 15 dB, so of course there is benefit to higher SNR. But Figure 3 enables one to consider whether the additional resources necessary to increase SNR above 15 dB should instead be allocated to accomplishing another task. Although higher SNR is always better, these resources must come from a finite resource budget – for example, the additional SNR might be achieved by integrating additional pulses. Figure 3 begs the question whether, at some point, these pulses should instead be allocated to detecting a different target (e.g., in a different area).

Another interesting point that emerges from consideration of this straightforward detection problem is whether the information gained from a single measurement can be carried forward to the next measurement. Most radar detection processors operate in a single shot paradigm, meaning that a test statistic is compared to a detection threshold. If the statistic exceeds the threshold, then a detection is declared. If not, then the statistic is thrown away and the procedure is repeated from scratch on future measurements. However, Figure 4 shows that in many cases, the probability of target presence has moved in the correct direction. Although it hasn't moved enough to declare a detection with confidence, there is still information that has been gained and shouldn't be discarded. Figure 4 shows distributions (obtained via histograms of simulation trial results) of the MI gained and the posterior target present probability. In generating these figures, we only used the target-present trials of the simulation, such that the ideal posterior probability of target presence is $\tilde{P}_1 = 1$, and the ideal information gained is 1 bit. First, we consider the left pair of figures, which corresponds to an SNR of 0 dB (i.e., they are a deeper look into the 0 dB results from Figures 2 and 3). These figures show that for many of the trials, the measurement obtained was not strong enough to move the target probability away from the prior probability of 0.5. When the probability doesn't change appreciably, this is essentially zero information gained, and we see from the far left plot that the most likely result of the measurements was to obtain little or no information. However, there is also a non-trivial number of times that the probability moved to the region around a target probability of 0.7 or 0.8. If the detection threshold is approximately 0.8 or 0.9, then this information should be retained and additional sensing resources could be allocated toward achieving an accurate decision. For the pair of figures on the right, the SNR is 5 dB, and we see that the results are further biased toward target presence. More information is gained more often, and we have many more trials indicating a possible target.

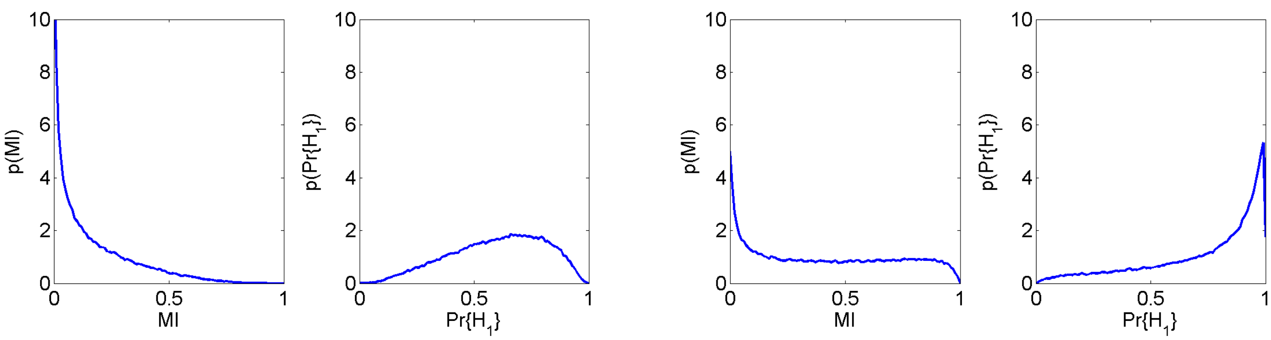

Figure 4: Distributions of information gained and posterior target present probability for a binary detection problem with SNR of 0 dB (left) and 5 dB (right).

These results shown in Figure 4 indicate the usefulness in taking the information gained from one set of measurements, and carrying it over to future measurements. In the case of cognitive radar, we wish to not only carry this information forward, but also to use this information in allocating sensing resources optimally for best sensing performance and efficiency.

3.2 Detection of a Signal with a Nuisance Parameter

A more interesting and practical problem involves detection of a signal that is parameterized by an unknown nuisance parameter. Examples of this type of detection problem include detection of a target with unknown range, detection of a target with unknown Doppler shift, and detection of a target in an unknown location. Despite the particular form that this problem may take, it is still a binary detection problem, and if the target's prior probability of being present is 0.5, then there is still only 1 bit of information to be acquired. The presence of one or more nuisance parameters certainly makes the detection problem more difficult, but they don't change the fundamental amount of information inherent in making a decision of target presence or absence.

Typical methods for dealing with nuisance parameters are the generalized likelihood ratio test (GLRT), and the Bayesian approach [3]. The type of approach depends on whether a pdf for the nuisance parameter is available. If no pdf is available, then the parameter can be treated as a deterministic unknown, for which the GLRT approach involves finding the maximum likelihood estimate of the unknown parameter followed by substitution into the likelihood ratio test. If a pdf is available, then the likelihood ratio test can be performed with the parameter's pdf incorporated into the distributions for the likelihood ratio test. For example, random phase can usually be assumed to follow a uniform pdf, and this pdf factors into the signal distributions in the signal-present and signal-absent hypotheses.

For our current analysis, we consider the case where we are trying to detect a target signal that could have one of N possible values of a nuisance parameter. Although nuisance parameters are usually continuous, using N discrete values of the nuisance parameter helps us to demonstrate the detection problem from the perspective of resource allocation. Hence, the target signal has a 50/50 chance of being present, and if it is present, then it is equally likely to have one of N possible parameter values. This detection problem can be expressed in our virtual source model according to

$$\mathbf{r} = \Theta\sqrt{E/N}\,\mathbf{s}_i + \mathbf{n} \tag{3}$$

where $\mathbf{r}$ is a length-N vector of detection statistics calculated for the N different parameter values, Θ is once again the binary random variable indicating target presence or absence, $\mathbf{s}_i$ is an indicator vector with all zeroes except for a single '1' in the location corresponding to the true value of the nuisance parameter (if signal is present), $\sqrt{E}$ is used to control the SNR of the scenario, and $\mathbf{n}$ is a vector of AWGN. The statistics in the elements of $\mathbf{r}$ can be used to compute conditional probabilities of target presence for the different nuisance parameter values. For example, let $A_1 \ldots A_N$ be the N different nuisance parameter values with prior probabilities of $\Pr\{A_1\}$, $\Pr\{A_2\}$, … $\Pr\{A_N\}$. It is then possible to compute the conditional probabilities of target presence (i.e., hypothesis H_1), which are $\Pr\{H_1|A_1\}$ through $\Pr\{H_1|A_N\}$, followed by the total probability of target presence according to

$$\Pr\{H_1\} = \Pr\{H_1 \mid A_1\}\Pr\{A_1\} + \Pr\{H_1 \mid A_2\}\Pr\{A_2\} + \ldots \Pr\{H_1 \mid A_N\}\Pr\{A_N\}\,. \tag{4}$$

Figure 5 shows mutual information (with a maximum of 1 bit) for this detection problem as a function of SNR for different measurement mechanisms. First, the solid blue line shows results for a single measurement defined according to (3) where each of the N nuisance parameter values is illuminated with the same intensity. Here, $\sqrt{E}$ is the total energy budget, such that each cell is illuminated with $1/N$th the total energy, and a single *measurement* is defined as a single capture of the length-N vector $\mathbf{r}$. For generating Figure 5, we set $N = 15$. A common rule of thumb is that SNR needs to reach approximately 10-12 dB to achieve somewhat reliable detection performance. In this example, because the energy is distributed evenly over 15 different nuisance parameters, the performance does not significantly improve until the SNR reaches 20-25 dB.

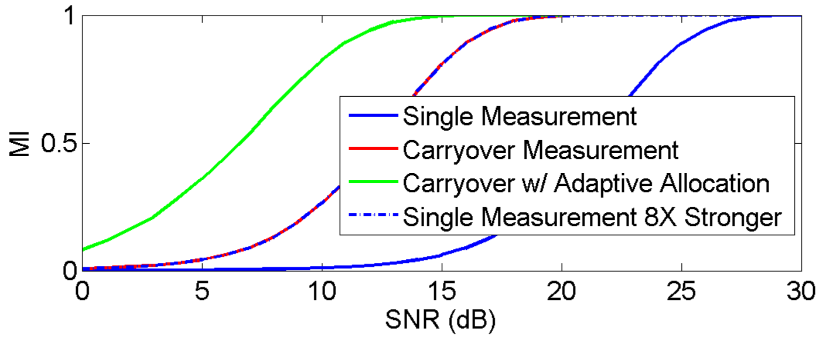

Figure 5: Mutual information for a detection problem with nuisance parameters.

However, as described in the previous section, in many cases the measurement is valuable for moving the probability of target presence in the correct direction, just not enough to make a reliable decision. Therefore, it would be helpful to carry over this information to a subsequent measurement, rather than make a decision based on unreliable data. For the current problem, this carryover could be achieved using Bayes' theorem to update the overall probability of target presence, as well as the probability of individual nuisance parameters, with each measurement received. Defining $\mathbf{r}_q$ as the measurement obtained on the qth iteration of the carryover measurement scheme, then the probability of target presence after collecting $\mathbf{r}_q$ is

$$\Pr\left\{\mathrm{H}_1^{(q)} \mid \mathbf{r}_q\right\} \propto \Pr\left\{\mathrm{H}_1^{(q-1)}\right\} \sum_{i=1}^{N} p\left(\mathbf{r}_q \mid \mathbf{s}_i, \mathrm{H}_1\right) \Pr\left\{\mathbf{s}_i\right\}. \tag{5}$$

In this way, data obtained can be carried over from one illumination to the next, thereby increasing effective SNR and improving performance. This strategy, which has the same general motivation as track-before-detection methods [4-5], leads to the performance seen in the red curve of Figure 5. Obviously, this carryover has improved the detection performance; but it is strictly a result of additional energy added to the problem. The number of measurement iterations before making a decision was $Q = 8$, and the performance curve has shifted to the left by $10\log_{10}(8) = 9$ dB. On the other hand, if a single measurement is taken, but that single measurement has 8× more energy allocated to it, then the performance is the same (blue dashed curve in Figure 5).

Finally, we consider what happens when we carry over information between measurements, but in a scenario that allows the different nuisance parameters to be non-uniformly illuminated. This capability obviously depends on the particular scenario – for example, it is usually not possible to change the illuminated energy from one range bin to the next. But if the nuisance parameter values indicated different locations in an illuminated scene, it might be possible to control energy allocation through the illumination pattern. In this case, we make a simple calculation according to

$$\frac{\sigma_n^2}{\Pr\left\{\mathbf{s}_i\right\}\left(1-\Pr\left\{\mathbf{s}_i\right\}\right)} \tag{6}$$

where σ_n^2 is the noise power in a single element of the detection statistic vector $\mathbf{r}$. When the probability of the ith nuisance parameter value being true is 0.5, this is a situation of maximum uncertainty, and the metric in (6) has a minimum value. On the other hand, when the probability of the ith nuisance parameter value being true is either 0.0 or 1.0, then this is a situation without *any* uncertainty, and the metric goes to infinity. We compute this metric for each of the N nuisance parameter values at each measurement iteration, and allocate energy according to a waterfilling solution on the metric. That is, cells with the lowest metric receive the strongest energy allocations because they are the most uncertain. Parameter values without any uncertainty do not need any energy allocation.

We can see in Figure 5 that this adaptive energy allocation provides another significant boost in performance. The first boost is obtained by the carryover of information. The second boost is obtained by allocating energy to those regions of the detection problem that have the most uncertainty, and phasing out illumination for those cells that have already been determined. While these concepts are being applied here to a fairly basic detection problem, the same principles apply to our additional work in adaptive beamsteering for target search and track functions, adaptive waveform parameter selection integrated with track-before-detect operation, and other novel radar applications of adaptive measurement.

Another method to compare the performance of the adaptive energy allocation, is to determine the average number of measurement iterations necessary to achieve a certain performance. For example, we have set a detection threshold at a probability of target presence equal to 0.98, and run many trials with a target present (but randomly located at different nuisance parameter values). We built statistics on the number of measurement iterations required to declare a detection, as a function of SNR and for equal energy allocation versus adaptive energy allocation. In Figure 6a (left), we show the distribution of iterations for an SNR of approximately 5 dB. The adaptive energy allocation (in red) clearly has a much higher incidence of needing fewer iterations to make a decision. Figure 6b shows the average number of iterations required as a function of SNR. At the lower end of SNR, adaptive allocation has a major benefit.

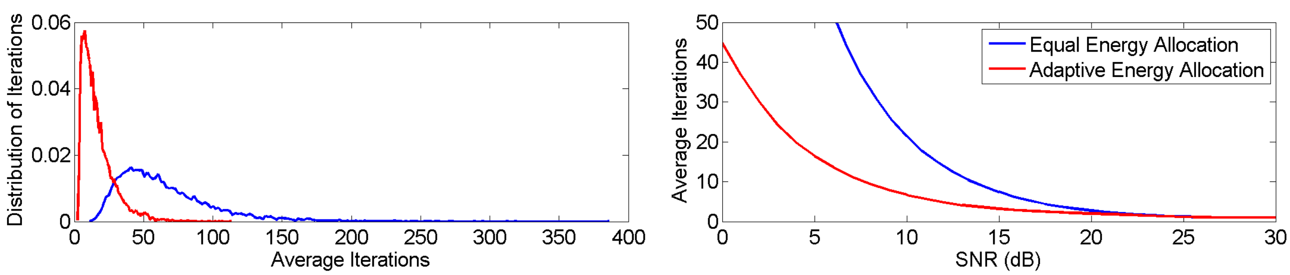

Figure 6: Statistics on number of measurement iterations to make a detection decision for adaptive energy allocation (red) and equal energy allocation (blue).

3.3 Detection of Multiple Targets with Nuisance Parameters

The previous scenarios involved a single target, if present. On the other hand, many radar applications require detection of multiple targets having different parameters such as range, Doppler shift, and/or location. In this section, we expand the previous concepts to detection of multiple targets, but where there exists some prior information on the number of targets in the scene.

We consider the previous example where there are N possible values of a nuisance parameter, but there may now be multiple targets that have different nuisance parameter values. The significant problem here is that keeping track of the potential scenario hypotheses becomes computationally cumbersome. For illustration purposes, consider a simplified scenario with $N = 5$ nuisance parameter values. If no targets are present, then there are no targets having any of the nuisance parameter values. If one target is present, then it could have the first nuisance parameter value, or the second, or the third, and so on. Figure 7 depicts the nature of the problem by explicitly listing out the different combinations that might be true for different number of targets. To continue along the theme of the previous section, we need to track the probability of being true for each of these possible states. One can imagine how this might become overwhelming for large state spaces and large numbers of targets, but in many cases the maximum number of targets can be reasonable.

Figure 7: Depiction of different nuisance parameter combinations that can result from different number of targets present in the scene.

In Section 3.2 where we assumed a maximum of a single target, this knowledge causes the hypotheses in different nuisance parameter cells to be correlated. For example, if a single target is likely to have a particular nuisance parameter value, then that necessarily means that other nuisance parameter values are less likely. Such knowledge allows resources to be focused on areas of highest uncertainty, while the information gained impacts the entire target parameter space. The same is true here in the current scenario, but the amount of correlation depends on the size of the target parameter space and the number of targets. In the limit as the number of targets is allowed to fill the entire parameter space, then gaining information about target presence in one cell has no impact on other cells, and resources (such as illuminated energy) cannot be focused so effectively.

Figure 8 demonstrates the possible behavior when the maximum number of targets is small compared to the resolvable target parameter space. In this example, there are six resolvable target parameter values, and the maximum number of targets in the scene is three. The prior probabilities on the number of targets are:

$$\Pr\{0 \text{ Targets}\} = 0.51; \quad \Pr\{1 \text{ Target}\} = 0.38; \quad \Pr\{2 \text{ Targets}\} = 0.1; \quad \Pr\{3 \text{ Targets}\} = 0.01.$$

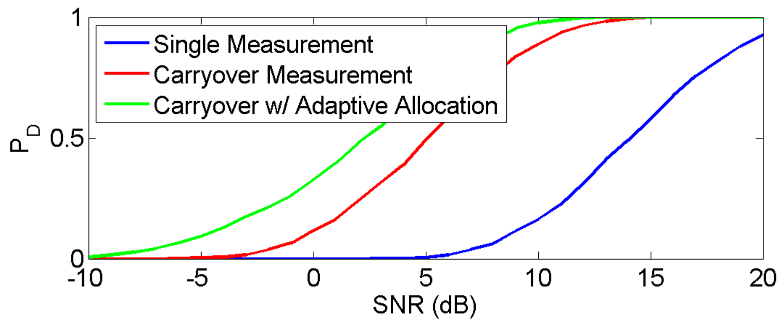

Figure 8: Probability of detection for 6 nuisance parameter cells and maximum of 3 targets.

For each trial in the simulation, the true number of targets is generated according to these probabilities. Once the number of targets is obtained, their parameter values are randomly generated as well. Each measurement comprises a detection statistic in each of the six resolvable cells, from which the probability of target presence in each cell and the expected number of targets present can be updated. Energy is allocated to these cells in a similar manner as before according to the probability of target presence in each cell at each measurement iteration. Once again, we observe the same themes – carryover of information provides a large performance improvement and adaptive energy allocation provides additional significant benefit. The mutual information curve (not shown) also follows the same trend as before.

4.0 MIXED TASKS

Section 3 demonstrates the potential benefit of assigning finite resources such as energy and timeline to achieve detection tasks. When there is uncertainty regarding certain parameters of the target, it may be possible to enhance performance by customizing illumination parameters to help clarify the most uncertain aspects of a given detection problem. However, the previous examples deal exclusively with a detection problem, which for a single detection decision has a maximum entropy of 1 bit. In other applications, it is often desirable to not only detect a target, but also to provide an estimate of its parameters; therefore, the parameter estimation problem should also be included as part of the sensing task.

Parameter estimation involves estimating a continuous parameter, which is a fundamentally different type of problem than detecting the correct hypothesis out of a finite set of alternatives. The differential entropy of continuous random variables is distinct from the entropy of discrete random variables [2]. The units of entropy are different, and different entropy values can be obtained depending on the units used for the parameter being estimated (i.e., metres versus kilometres). These factors make it difficult to define optimization criteria for mixed tasks such as detection and estimation. In terms of estimating multiple parameters having different units (e.g., metres for range and Hertz for Doppler), one might normalize the units in terms of fundamental resolution cells. But for performing both detection and estimation operations, these heterogeneous quantities cause some difficulty in combining to obtain an objective metric.

Further complicating the situation is that the best measurement setup for detection may not be the best setup for estimation. We consider here, for example, the mixed tasks of detection and range estimation, and we demonstrate that the optimum waveforms may be different for the two tasks. To perform a demonstration, we synthesized an extended target by randomly generating scatterers over a 12-meter extent. Once these scatterers were generated, we then assumed that the resulting target impulse response (the sum of reflections at the different delays, amplitudes, and phases) was fixed and known. The coherent interactions of the scatterers varies with frequency and can be destructive, constructive, or somewhere in between. Figure 9 shows the (complex baseband) frequency-selective target behaviour that results from such a multi-scattering object. We then modelled a scenario where this target might or might not be present, but if it was present, it had a random range to the front of the target that varied uniformly from 0 to 15 meters. We considered two waveforms having a duration of 0.16 µs. The first was a pure sinusoid matched in frequency [6] to the peak target response (at approximately 390 MHz in Figure 9). The second was a 100-MHz waveform centered at 0 MHz (in complex baseband spectrum). Figure 10 shows the SNR achieved at the output of the optimum filter for both waveforms as a function of the input signal energy. As expected, output SNR grows with input signal energy, but we also see that the matched sinusoid provides an SNR gain of about 7 dB compared to the 100-MHz waveform.

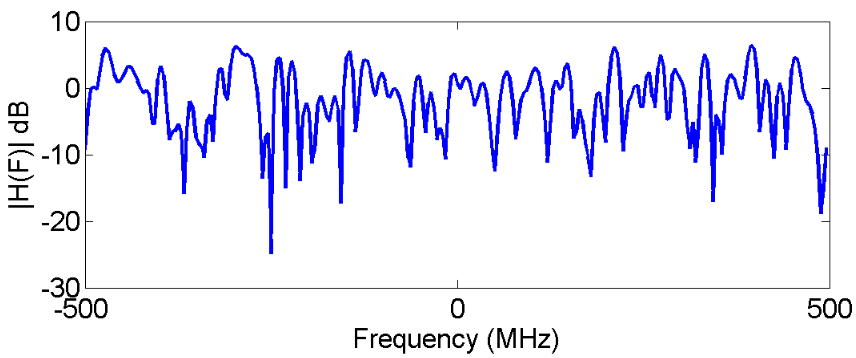

Figure 9: Frequency selective target RCS versus frequency.

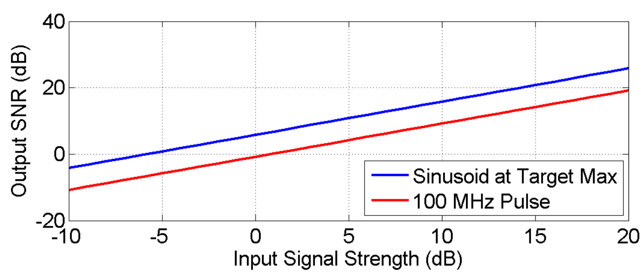

Figure 10: Input/Output SNR relationship for a frequency-selective target and two different waveforms.

Because SNR is critical for detection, we see in the left panel of Figure 11 that the matched sinusoid waveform provides better detection performance (reduced probability of miss). In the high-SNR asymptotic region, the two curves have a relative shift of approximately 7 dB, which corresponds to the difference in SNR obtained by matching the sinusoidal waveform to the peak frequency response of the target. However, the right panel of Figure 11 shows the mean-squared error in the range estimation (obtained through Monte Carlo simulation) for detected targets. Below about 10 dB SNR, the range estimate is unreliable, but above this threshold SNR the performance of the 100-MHz waveform improves dramatically compared to the narrowband tone waveform. Of course this behaviour is due to the fact that range estimation is intimately related to range resolution, which depends directly on bandwidth. In fact, the Cramer-Rao Bound [7] for range resolution improves linearly with SNR, but quadratically with bandwidth. Therefore, we must conclude that the matched narrowband waveform is best for detection, while the waveform with wider bandwidth is best for range estimation. Consequently, it is impossible to optimize a waveform for both, and some trade-off or compromise will be necessary.

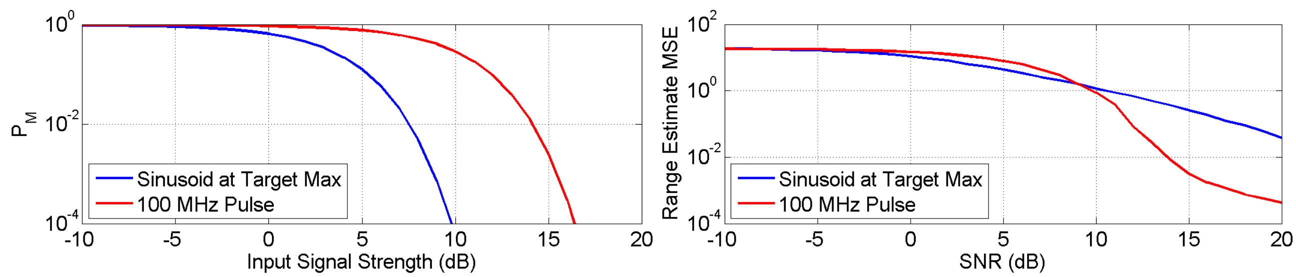

Figure 11: (Left) Probability of miss and (Right) Range MSE for extended target and two different waveforms.

5.0 COMMENTS ON POSSIBLE APPLICATIONS

While more details on applications of the above concepts are provided in our second paper for this lecture series (titled “Cognitive Radar Networks and Integrated Radar Modes” [8]), here we give a few brief comments regarding some potential applications of the resource allocation philosophy demonstrated above through fundamental detection and estimation problems.

The basic principles outlined above have been applied to problems of waveform design for target recognition, adaptive parameter selection for radar search-and-track modes, and integration of adaptive parameter selection with track-before-detect methods. For the target recognition application, the fundamental task is to identify the target type or class, which is encoded with a discrete random source variable. This task is typically made more complicated by nuisance parameters such as target orientation. In past work, we have divided nuisance parameters such as orientation and target range into small local regions, assigned a probability of the target parameters being within this region, and modelled the signals conditioned on each nuisance parameter region as Gaussian. The prior probabilities for each nuisance parameter region might be obtained, for example, from previous track information. Once these probabilistic models are defined from target template libraries and priors on the nuisance parameters, we can design a waveform that enhances discrimination between the different target types. With each transmission and measurement, the nuisance parameter probabilities and individual target type probabilities can be updated, such that a new optimized waveform can be designed for the next transmission. With such a method, we have shown improved performance for a fixed number of transmissions, or reduced number of transmissions to achieve a fixed performance goal [9]. These conclusions are similar to those made in Section 3.2.

For the search-and-track application, we have modelled the target parameter space as a grid of resolution cells, each with a probability of target presence. These probabilities are updated with each illumination, or allowed to regress back to a steady state value when not illuminated for several iterations. By tracking probabilities from illumination to illumination, we can adaptively select waveform pulse repetition intervals [10] and/or beamsteering locations [11] in order to improve the chances of detecting weak targets moving through a scene. To balance between target detection and tracking (mixed tasks), we use a scale factor to linearly combine detection-related entropy with parameter estimation-related entropy. By allowing non-illuminated cells to converge back to a quiescent state, we introduce dynamics into the scene. These various features can be combined to weight the relative importance of the detection and tracking tasks, to assign target-specific importance via variable weight factors, and to apply terrain-specific dynamics that model the likelihood of targets appearing or disappearing in different areas in the scene. For example, we can code the dynamics of the scene such that known roads and open areas are searched more frequently than mountainous or shadowed regions. Many beneficial implementations are possible, but at the admitted expense of additional computation to keep track of the many different permutations of the possible radar environment and to quickly compute optimum illumination parameters. In addition to the grid of probability cells, we have also used a multi-target track-before-detect approach in [10].

Again, more details are given in [9], along with our conclusions regarding potential benefits and pitfalls of different implementations.

6.0 CONCLUSIONS

The purpose of this paper was to demonstrate through textbook-type examples how adaptive and optimized allocation of sensing resources can improve sensor performance. These resources, such as power and timeline, are always constrained in any sensor system. It is possible to achieve more efficient allocation of these resources, but to do so requires task-specific performance metrics and sensing models parameterized by the variables of interest. In the case of multiple tasks, especially tasks of heterogeneous type such as detection and estimation, the sensor optimization for the different tasks might conflict. In such cases, a compromise is necessary. It may even be the case that measurements optimized for one tasks may be completely useless for another; therefore, there is a natural trade-off between enhanced performance through specifically optimized measurements and the robustness of general-purpose measurements.

There are many potential applications and benefits, but each case requires a representation of what is known, unknown, and the desired tasks. Hopefully, this paper encourages the reader to consider a radar system's ultimate exploitation tasks, and to consider whether existing and/or future radar systems can be used in more flexible ways to perform these tasks more effectively.

REFERENCES

[1] M.A. Neifeld, A. Ashok, and P.K. Baheti, "Task-specific information for imaging system analysis," *Journal of the Optical Society of America*, vol. 24, no. 12, pp. B25-B164, 2007.

[2] T.M. Cover, and J.A. Thomas, *Elements of Information Theory*, 2nd edition. Hoboken, NJ: Wiley, 2006.

[3] S.M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*. Upper Saddle River, NJ: Prentice-Hall, 1998.

[4] M.S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A tutorial on particle filters for online nonlinear non-Gaussian Bayesian tracking," *IEEE Transactions on Signal Processing*, vol. 50, No. 2, pp. 174-188, Feb. 2002.

[5] M.G. Rutten, N.J. Gordon, and S. Maskell, "Efficient particle-based track-before-detect in Rayleigh noise," *in Fusion 2004: Proc. 7th Int. Conf. on Information Fusion*, Stockhelm, Sweden, June 2004.

[6] M.R. Bell, "Information theory and radar waveform design," *IEEE Trans. Inf. Theory*, vol. 39, no. 5, pp. 1578–1597, 1993.

[7] S.M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Upper Saddle River, NJ: Prentice-Hall, 1993.

[8] N.A. Goodman, "Cognitive radar networks and integrated radar modes," *NATO SET-216 Lecture Series on Cognition and Radar Sensing*, 2015.

[9] N.A. Goodman, "Waveform design for radar target recognition with GMM-based Classifier," in *Proc. NATO SET-204 Specialists' Meeting on Waveform Diversity*, Sept. 2014.

[10] J. Bae and N.A. Goodman, "Adaptive PRF selection technique for multiple targets in track-before-detect," in *Proc. 5th International Workshop on Computational Advances in Multi-Sensor Adaptive Processing*, St. Martin, pp. 448-451, Dec. 2013.

[11] R.A. Romero and N.A. Goodman, "Cognitive radar network: cooperative adaptive beamsteering for integrated search-and-track application," *IEEE Trans. Aerospace and Electronic Systems*, vol. 49, no. 2, pp. 915-931, April 2013.